Name of Course	: **B.A. (Prog.) DSE: Mathematics**
Unique Paper Code	: **62357502**
Name of Paper	: **Differential Equations**
Semester	: **V**
Duration	: **3 hours**
Maximum Marks	: **75 Marks**

Attempt any four questions. All questions carry equal marks.

1. Solve the differential equation

$$(x^2 + y^2 + 1)\,dx - 2xy\,dy = 0.$$

Also, solve the differential equation

$$(D^2 + 4)y = \sin 3x + e^x$$

where $D \equiv \frac{d}{dx}$.

2. Show that $y_1(x) = \sin x$ and $y_2(x) = \sin x - \cos x$ are linearly independent solutions of

$$y'' + y = 0.$$

Determine c_1 and c_2 so that the solution

$$\sin x + 3\cos x \equiv c_1 y_1(x) + c_2 y_2(x).$$

Also, solve

$$x^2y'' - 2xy' + 2y = 6, y(0) = 3, y'(0) = 1.$$

3. Using the method of variation of parameters ,solve

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = x^2e^x.$$

Also, solve

$$yz^2dx - xz^2dy - (2xyz + x^2)dz = 0.$$

4. Solve the equations:

$$\frac{dx}{z(x+y)} = \frac{dy}{z(x-y)} = \frac{dz}{x^2+y^2}.$$

Also, solve

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - 3y = x + \sin x.$$

5. Form a partial differential equation by eliminating the function f from $z = f\left(\frac{y}{x}\right)$.

Also solve the partial differential equation

$$\left(\frac{y^2 z}{x}\right) p + xzq = y^2.$$

6. Classify the partial differential equation into elliptic, parabolic or hyperbolic

$$x(xy-1)r - (x^2y^2-1)s + y(xy-1)t + (x-1)p + (y-1)q = 0$$

where $r = \frac{\partial^2 z}{\partial x^2}$, $s = \frac{\partial^2 z}{\partial x \partial y}$, $t = \frac{\partial^2 z}{\partial y^2}$, $p = \frac{\partial z}{\partial x}$, $q = \frac{\partial z}{\partial y}$.

Also, find a complete integral of partial differential equation

$$z = px + qy + p^2 + q^2.$$